Teaching and learning approaches

Guidance on teaching and learning approaches is presented in three sections:

- ‘Some principles for effective learning’, based on the 2008 programme of study in mathematics
- ‘Some principles for effective teaching’, based on research over many years into the teaching of mathematics
- ‘Further support to develop pedagogy and practice’, which references existing Strategy guidance on lesson design, teaching repertoire, etc.

Some principles for effective learning

This section is informed by the curriculum aims of the 2008 programme of study. By synthesising and interpreting the aims, key processes, key concepts and curriculum opportunities the intention is to provide a supportive reference paper which the whole department can use to reflect on priorities for development in teaching and learning and so phase the implementation of the new programme of study.

Pupils learn about and learn through the key mathematical processes

Key processes need to be experienced as components of a whole cycle and this can be reflected within a single lesson as well as through a unit of work. Investigative and problem-solving opportunities should be designed so that pupils cycle through the processes several times and also move backwards and forwards between the stages as ideas mature, modify and change. In this way the notion of a cycle provides a helpful structure but does not become restrictive.
The diagram represents the dual nature of mathematics, both as a tool for solving problems in a wide range of contexts and as a discipline with a distinctive and rigorous structure. So pupils become successful learners by developing competence in applying mathematics effectively in a range of contexts, including those from within mathematics itself. There are two ways of thinking about pupils’ experience of the key mathematical processes that lie at the heart of the revised programme of study.

They need opportunities to **learn about** the mathematical processes and to reflect on how they are improving in these skills. This could include designing lessons or units where there is no new content and the focus is on improving the process skills. They need opportunities to **learn through** using the mathematical processes. As pupils gain confidence in the skills of applying these processes they can use them to develop their understanding of topics within the range and content of the curriculum.

### Pupils work collaboratively and engage in mathematical talk

It is through paired and group work that pupils gain confidence in their ability to communicate mathematics effectively. Choosing a rich task will usually provide pupils with the chance to explain and justify, question and disagree. Over time the level of dialogue in the classroom becomes more mathematically rich as pupils pose questions to each other and develop more convincing arguments orally. As this kind of dialogue becomes a regular part of their work on mathematics pupils are forced to think in this way, preparing questions for one another and rehearsing arguments. We could describe this as developing a habit of ‘self-talk’; that is they are naturally developing the thinking which will support more independent work.

### Pupils work on sequences of tasks

Within the planning and teaching of units of work there need to be sequences of lessons which do not move too quickly from one topic to another or from one task to another. Instead, pupils need to be provided with a sequence of learning which is planned to become more challenging within a phase of a unit. One way of doing this is to select a task or sequence of related tasks which develop over a number of lessons. This has the advantage of reducing the burden of producing and introducing different tasks in each lesson. It means that more of a lesson is dedicated to pupils actively doing mathematics rather than listening to instructions for new and different topics and tasks. Most importantly a sequence of tasks, involving the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts makes mathematical progression more explicit to the pupils. In this way pupils develop the competence to apply suitable mathematics accurately within the classroom and beyond.

### Pupils select the mathematics to use

Pupils can begin to see the power and purpose of their mathematical learning when they are given the opportunity to make decisions about the mathematical tools (including ICT) to help them to solve a problem or investigate from a given stimulus. As pupils use existing mathematical knowledge to investigate or create solutions to unfamiliar problems their confidence increases and they come to see that doing mathematics is an interesting and enjoyable activity. They are then more likely to apply mathematical skills in life effectively, in their wider studies and ultimately in employment. Unit plans adapted to meet the new curriculum should build in a variety of open and closed tasks, ensuring that the contexts for some task are real and others are abstract. In this way pupils come to appreciate mathematics for itself as well as understanding that it is used as a tool in a wide range of contexts.
Pupils tackle relevant contexts beyond the mathematics classroom

In order for pupils to be functional in mathematics and motivated to take their learning further they need to hone their knowledge, skills and understanding by applying suitable mathematics accurately within the classroom and beyond. This means planning units where pupils are not learning new content but are working on problems that arise in other subjects and in contexts beyond the school, such as architecture or engineering. In many cases a solution will involve using mathematics as a model to interpret or represent situations. Applications involving modelling changes in society and the environment or managing risk (for example, insurance and investments) could be used to stimulate discussion about important issues such as financial capability or environmental dilemmas. The assumptions and simplifications involved in the process of modelling a real context should be made explicit so that pupils come to realise that mathematics itself is essentially abstract and that a model or representation has limitations to its scope.

Pupils are exposed to the historical and cultural roots of mathematics

If they are given the chance to learn about problems from the past that led to the development of particular areas of mathematics, pupils can begin to appreciate that people of all cultures use mathematics to make sense of the world around them. They may be fascinated to find out that pure mathematical findings sometimes precede practical applications, and their curiosity may be aroused to think that mathematics continues to develop and evolve. This will engage and motivate pupils to become more aware of the nature of mathematics and of the mathematics around them.

Some principles for effective teaching

Research shows that the following principles underlie effective teaching. They are based on those included in Improving learning in mathematics: challenges and strategies, by Malcolm Swan University of Nottingham, in the ‘Standards Unit box’ (Improving learning in mathematics, The Standards Unit, DfES1599-2005DOC-EN). The list is provided to support evaluation of current teaching approaches and to stimulate departmental discussions about improving the effectiveness of current teaching.

Build on the knowledge pupils bring to a sequence of lessons

Design activities which uncover prior learning and offer pupils opportunities to express their understanding. For example:

- pose a problem to the whole class to stimulate paired discussion and to set the agenda for the next few lessons
- set up pairs or groups to draw and share a concept map or equivalent diagram showing the interconnections of existing understanding. Revisit the ‘maps’ and add to them as the learning emerges throughout the unit.

The following Strategy resources in the Secondary mathematics planning toolkit may help with this approach (Rich tasks folder):

- Leading in learning (KS3 and KS4 training materials and exemplification in mathematics)
- Bridging plans: from KS3 to KS4
- Interacting with mathematics in Key Stage 3 – proportional reasoning
  - Year 7 Fractions and ratio minipack and resources, especially the key lesson
Expose and discuss common misconceptions

Pupils make mistakes for a variety of reasons. Some are due to lapses in concentration, hasty reasoning, memory overload or a failure to notice important features of a problem. Others, however, are symptoms of more profound mathematical difficulties. Where mistakes are the result of consistent, alternative interpretations of mathematical ideas we refer to them as misconceptions. These should not be dismissed as ‘wrong thinking’ as they may be necessary stages of conceptual development. Design activities so that misconceptions are systematically exposed by allowing time in the lessons for pupils to reflect and discuss these conflicts. For example:

- ask pupils to complete a task, using more than one method, and then to resolve conflicting answers
- present statements to be classified and justified as always true, sometimes true or never true.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach:

- *Misconceptions in mathematics* (Pedagogy folder, Improving subject knowledge sub-folder)
- *Teaching mental mathematics from level 5*
  - *Shape and space* (Rich tasks folder)

You might also find useful, as a separately available CD-ROM including video:

- *Mathematics: developing dialogue and reasoning* (DfES 00243-2006CDO-EN)

Develop effective questioning

Aim to invite a range of responses to your questions by asking more open and probing questions which promote higher-level reflective thinking. Allow time for pupils to think before offering help or moving on to ask someone else and allow time for yourself so that you think about your response. For example:

- establish a routine through which pupils share their answers in pairs before you take whole-class feedback. This ‘pair/share’ not only builds confidence, it also increases the number of pupils who feel that their response has been heard.
- be explicit about types of questions you use and encourage the pupils to use the same types of question. A display of question stems can be helpful for you and the pupils:
  - What if…?
  - Why do you think…?
  - When would it not work…?
  - How do you know…?

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach (Pedagogy folder):

- Standards Unit *Improving learning in mathematics* (Pedagogy folder)
- *Pedagogy and practice: Teaching and learning in secondary schools*
  - *Unit 7 questioning* (Pedagogy folder)
Also, as separately available CD-ROMs:

- Assessing pupils’ progress in mathematics at Key Stage 3, probing questions (DfES 00007-2007CDO-EN)
- Mathematics: developing dialogue and reasoning (DfES 00243-2006CDO-EN)

**Use cooperative small-group work**

Ensure that everyone is confident and benefits from participating in discussions by designing tasks which require collaboration in pairs or small groups and establish this as a regular feature of mathematics lessons. For example:

- use classification activities with only one set of objects per pair or group so that joint decisions have to be made
- ask pupils to create a spider diagram of connections with one large sheet of paper for three pupils.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach:

- *Pedagogy and practice: Teaching and learning in secondary schools*
  - Unit 10 group work (Pedagogy folder)
- *Interacting with mathematics in Key Stage 3 – handling data*
  - *Y8 handling data minipack* (Rich tasks folder)

**Emphasise methods rather than answers**

Focus on pupils developing their repertoire of appropriate methods rather than on obtaining correct answers to a long list of similar problems. This is likely to involve aiming to work on fewer problems than would appear in a typical textbook exercise. For example:

- direct pupils to tackle the same problem, using more than one method, and work in pairs to compare solutions and evaluate their efficiency
- ask pupils to redesign a problem so that it is more challenging or simpler and give it to their group to solve.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach.

- *Interacting with mathematics in Key Stage 3* (Rich tasks folder)
  - *Handling data* (Wise words and other tasks)
  - *Proportional reasoning* (Year 8 multiplicative relationships, Year 9 proportional reasoning, Enhancing PR in Year 8 and Year 9)

**Use rich collaborative tasks**

Think about how to design tasks that motivate a need to learn and encourage the pupils to be creative, curious and reflective. Pupils’ mathematical thinking will be improved if they have to make decisions and ask questions. The learning is made memorable when pupils enjoy the tasks and are surprised by outcomes. Richer tasks allow all learners to find something challenging and at an appropriate level to work on. Examples of accessible and extendable tasks can be developed from ‘routine’ tasks by changing the initial stimulus and the questions asked. For example:

- help pupils to consolidate their understanding of algebraic factorisation, expansion and simplification by working with jigsaw or domino cards showing matching expressions. Extend this to include their own design of such cards
• ask two groups of pupils to debate opposing arguments which support or refute a hypothesis where data is supplied in a spreadsheet.

The following resources in the Secondary mathematics planning toolkit may help with this approach (Rich tasks folder):

  • *Teaching mental mathematics from level 5*
  • Standards Unit *Improving learning in mathematics*

**Create connections between mathematical topics**

Design activities for existing units which make explicit connections within and across mathematical topics. For example:

• matching tasks which require pupils to recognise different representations of the same mathematical idea.

You may also plan to include more cross-strand units to develop stronger connections. For example:

• a functional mathematics unit presenting a real context requiring exploration or investigation. In such units pupils could work on a range of mathematical connections needed to reach a resolution.

The following resources in the *Secondary mathematics planning toolkit* may help with this approach (Rich tasks folder):

  • Standards Unit *Improving learning in mathematics*
  • *Teaching mental mathematics from level 5*
    • *Measures and mensuration booklets*
  • *Interacting with mathematics in KS3 – proportional reasoning*
    • *Year 9 proportional reasoning*

**Use technology in appropriate ways**

Present mathematical concepts in dynamic, visually exciting ways that engage and motivate learners. Introduce, explore and represent concepts, structures and processes in new and revealing ways. Often dynamic images will permit insights and understandings which are difficult to convey in other ways. For example:

• Display an equation of the form $y = mx + c$ on the same screen as the associated table and graph in order to explore the relationship between them

• Explore a dynamic diagram showing how the angle formed between two straight lines changes as the lines move. Extend to parallel lines and an intercept.

The following resources from the *Secondary mathematics planning toolkit* may help with this approach (Rich tasks folder):

  • *Interacting with mathematics in Key Stage 3 – proportional reasoning*
    • *Year 7 fractions and ratio*, interactive teaching programmes
  • *ICT in mathematics*, ICT lesson plans
Further support to develop pedagogy and practice

The 2001 Framework for teaching mathematics: Years 7, 8 and 9 established several principles which teachers found useful in guiding their planning. They included dimensions of a teaching repertoire such as modelling, questioning and explaining and aspects of lesson design such as structuring learning and starters and plenaries.

As further support for these developments the Pedagogy and Practice materials were published a few years later.

Pedagogy and Practice: Teaching and learning in secondary schools is often referred to as ‘the ped pack’. It is a suite of study guides created to support the professional development of teachers across all subjects in secondary schools. They provide guidance on the relationship between pedagogic approaches (teaching models), teaching strategies, techniques and methods of creating the conditions for learning in order to inform lesson design. The techniques suggested in each study guide are tried and tested and draw on both academic research and the experience of practising teachers.

Many teachers, who began to work on new teaching strategies from the initial, brief guidance in the Framework, moved on to more detailed developments through the ‘ped pack’ guidance. For example, the structuring learning booklet elaborates the original Framework guidance on structured lessons. It describes dividing lessons into a series of episodes, choosing from a range of strategies and techniques to motivate pupils and examines three pedagogic approaches – direct interactive, inductive and exploratory – to show how they can help pupils develop tools for learning, such as inductive thinking or enquiry skills.

The full list of booklets is given below, but you are unlikely to require the entire set at any one time. Instead, think about what support you need and consider downloading one or two booklets; most are only 24 pages and can be accessed at www.standards.dcsf.gov.uk/secondary/keystage3/all/respub/sec ppt10.
Designing lessons
Unit 1 Structuring learning
Unit 2 Teaching models
Unit 3 Lesson design for lower attainers
Unit 4 Lesson design for inclusion
Unit 5 Starters and plenaries

Teaching repertoire
Unit 6 Modelling
Unit 7 Questioning
Unit 8 Explaining
Unit 9 Guided learning
Unit 10 Group work
Unit 11 Active engagement techniques

Creating effective learners
Unit 12 Assessment for learning
Unit 13 Developing reading
Unit 14 Developing writing
Unit 15 Using ICT to enhance learning
Unit 16 Developing effective learners

Creating conditions for learning
Unit 17 Improving the climate for learning
Unit 18 Learning styles
Unit 19 Classroom management