Guidance note
Algebra – sequences, functions and graphs

This material has been written to support a mathematics teacher in a mainstream classroom. The context is an algebra unit which might address some or all of these objectives:

- describe integer sequences; generate terms of a simple sequence, given a rule (Y7)
- generate sequences from patterns or practical contexts and describe the general term in simple cases; represent them in mappings (Y7)
- express simple functions algebraically and represent them in mappings or on a spreadsheet (Y8)
- generate points in all four quadrants and plot the graphs of linear functions, where $y$ is given explicitly in terms of $x$, on paper and using ICT; recognise that equations of the form $y = mx + c$ correspond to straight line graphs (Y8)

All pupils have an entitlement to explore these ideas.

Pupils working at level 3 and below can access these relatively sophisticated concepts and make links between them if they are given the resources and activities to support their thinking and developing understanding.

Possible barriers to progress can be tackled using approaches such as those described below. These materials offer a starting point and are not the only way to address the difficulties highlighted here. Teachers know their pupils and will bring to these suggestions their own ideas and creative approaches.

The guidance is presented in a consistent format, describing the potential barrier to progress, and then offering ideas for teaching and questions which can support the suggested pupil activities and opportunities for mathematical talk. Finally, supporting resources are listed and a range of relevant assessment criteria is shown.

It should be noted that where primary resources and support are referenced content will need to be adapted to ensure age-appropriateness.
Algebra objective

*describe integer sequences; generate terms of a simple sequence, given a rule (Y7)*

Possible barrier

Pupils cannot describe or recognise sequences.

Note: Do not allow issues with counting to become an obstacle. Pupils operating at level 1 or below will have problems with place value but can still access the learning objective above.

### Teaching ideas and questions

Give a practical demonstration of a sequence of objects or shapes in a line, such as beads in a necklace or bracelet, or a pattern in context, such as a repeating pattern on a fabric. Ask pupils to continue the sequence and explain their choices (using beads or pictures).

- *Why doesn’t this shape, blue bead, etc. continue the pattern?*

Cover up parts of the sequence and ask:

- *What goes here?*
- *Describe this sequence. What can you see?*

Give a practical demonstration of a sequence of sounds, for example, drumbeats or clapping a sequence of sounds, with some difference in rhythm.

- *Explain how this sequence could be continued.*

### Pupil activities and opportunities for mathematical talk

Ask pupils to extend sequences and identify missing elements. Count aloud or list a sequence which includes a deliberate mistake; seek pupils’ responses.

Use a variety of contexts, such as:

- patterns of pictures
  - patterns in fabrics
  - beads in necklaces or bracelets
  - coloured shapes in a row
  - letter sequences
- colour sequences
  - using PowerPoint
  - using flash cards
- musical sequences
  - clapping sequences
  - sequences of musical notes
  - sequences of words in simple songs and raps
- objects in a set
  - sticks of interlinking cubes or centimetre cubes if motor skills are strong
  - objects in a bag, in a drawn ring etc.
### Supporting references/resources

- coloured beads and laces
- fabric samples, some with repeating patterns
- letter tiles
- interlinking cubes
- centimetre cubes
- modelling clay
- hundred squares
- blank number lines
- coloured card cut into rectangles for flash cards
- a simple keyboard or xylophone

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<tr>
<th>P8</th>
<th>L2</th>
<th>L3</th>
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<tbody>
<tr>
<td>talk about, recognise and copy simple repeating patterns and sequences</td>
<td>recognise sequences of numbers, including odd and even numbers</td>
<td>recognise a wider range of sequences</td>
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</table>
Algebra objective

*generate sequences from patterns or practical contexts and describe the general term in simple cases (Y7)*

Possible barrier

Pupils cannot generate sequences.

Note: Pupils who have a better grasp of number may deduce rules linking numbers where no linear relationship is present, such as 1, 2, 4, …

### Teaching ideas and questions

Ask pupils to continue sequences that increase in twos or tens.

- *Show me a sequence that has 8 in it. Convince me that you are right.*
- *Why is 17 not in the sequence 2, 4, 6, 8, …?*
- *Look at these two sequences:
  1, 3, 5, 7, …
  2, 4, 6, 8, …
  What is different about these sequences? What is the same about these sequences?*

Extend this to include sequences that increase or decrease in ones, twos, fives and tens.

- *Show me a sequence that has 10 in it. Convince me that you are right.*
- *Why is 27 not in the sequence 5, 10, 15, 20, …?*
- *Look at these two sequences:
  10, 20, 30, 40, …
  90, 80, 70, 60, …
  What is the same about these sequences? What is different about these sequences?*

### Pupil activities and opportunities for mathematical talk

Provide a variety of sequences for pupils to discuss and agree on properties. Ask them to decide on how to extend given sequences and articulate how they know when elements are missing.

Support can be offered by:

- having calculators available so that pupils with difficulties in counting are not excluded from developing their understanding of sequences and recognition of number patterns
- continuing to use concrete apparatus such as interlinking cubes
- recording number sequences on number lines as hops of constant size
- using the constant function on a calculator to generate sequences.
### Supporting references/resources

- coloured beads and laces
- letter tiles
- coloured card cut into rectangles for flash cards.
- a simple keyboard or xylophone
- centimetre cubes
- interlinking cubes
- hundred squares
- blank number lines

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<td>talk about, recognise and copy simple</td>
<td>recognise sequences of numbers, including odd and even</td>
<td>predict what comes next in a simple number,</td>
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<tr>
<td>repeating patterns and sequences</td>
<td>numbers</td>
<td>shape or spatial pattern or sequence and</td>
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<td>give reasons for their opinions</td>
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</table>
Algebra objective  
*express simple functions algebraically and represent them in mappings or on a spreadsheet (Y8)*

Possible barrier  
Pupils do not recognise that a symbol can be used to represent a range of values and that we can choose which values these take.

### Teaching ideas and questions

Say to pupils:

*John thinks of a number and so do I. Our numbers are the same. Are the following sometimes, always, or never true? Explain how you know.*

- *When I add 5 to both numbers, the new numbers are the same.*
- *When I double both numbers, the new numbers are the same.*
- *If I add 2 to John’s number and take 2 from my number, my number is bigger.*

Repeat where the two initial numbers have a difference of 2.

Go on to use symbols:

*Look at this: ♠ + □ = 10*

- *What could ♠ and □ represent here? Are these the only possible answers? How do you know?*

### Pupil activities and opportunities for mathematical talk

Ask pupils to suggest some symbols that we use in mathematics and group them as follows:

- symbols that represent numbers (such as 6, π, a as part of an equation such as \(a + 3 = 5\))
- symbols that represent operations (such as +, √, ×)
- symbols that represent relationships (such as =, ≥)
- symbols that represent units of measurement (such as m, kg).

Ask pupils to use their own symbols for the digits 1–4. How would they use these to show the numbers 7, 14 or 12?
Küchemann (1981) describes six different ways that pupils interpret and use letters:

(i) Letter evaluated
   The letter is assigned a numerical value from the outset (that is, \( a = 1 \), \( b = 2 \), etc.).

(ii) Letter not used
   The letter is ignored or its existence is acknowledged but without giving it a meaning.

(iii) Letter used as an object or label
   The letter is regarded as a shorthand for an object (\( a \) is for apple).

(iv) Letter used as a specific unknown
   The letter is used as a specific but unknown number.

(v) Letter used as a generalised number
   The letter is seen as representing several (integer) values.

(vi) Letter used as a variable
   The letter is seen as representing a range of unspecified values.

It has been long recognised that the common usage of the letter \( x \) leads to further confusion as pupils mistake it for the multiplication symbol.

We are seeking to avoid reinforcing the first three which are misconceptions.


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<td>L1</td>
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<td>L3</td>
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<tr>
<td>recognise and use a simple pattern or relationship</td>
<td>begin to represent their work using symbols and simple diagrams</td>
<td>begin to understand the role of ‘=’ (the ‘equals’ sign)</td>
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</table>
Algebra objective  
*express simple functions algebraically and represent them in mappings or on a spreadsheet (Y8)*

Possible barrier  
Pupils cannot generate the dependent variable by substituting integer values for the independent variable.

Note: Initially, the rules should use addition and subtraction.

### Teaching ideas and questions

Ask pupils:
- *Your number is always 2 more than my number.*  
  What is your number if my number is 2, 5, etc.?
- *To help remember the rule I will use these symbols to represent the connection:*  
  - What does the + mean?  
  - What does the 2 mean?  
  - What does the = mean? (emphasise ‘is the same as’ rather than ‘makes’.)
  - What does the \( \square \) represent?  
  - What does the \( \text{} \) represent?
- *I give you a number. You have to …*  
  …add 20, …make it up to 10, …double it, …double it and add 5  
  What do you get if the number I give you is …?

### Pupil activities and opportunities for mathematical talk

In order to reduce potential arithmetical barriers, you may choose to restrict the rules used to those of the form \( y = x \pm c \).

Since arithmetic is not the area being developed, allow pupils to use calculators for even the simplest cases if they find this supportive.

Base the work around pupils’ own suggestions for naming the variables (see the previous barrier for this objective for further information).

Function machines and mapping diagrams, including software to animate these, make a useful context, enabling discussion on inverse operations.

Focus pupils on the way the calculations change when the value of the independent variable changes rather than the answer, i.e. \( \square = \square + 2 \) becomes \( \square = 3 + 2 \) when \( \square = 3 \), etc. Ask them to comment on the pattern they notice in the way the calculations look as different numbers are substituted for \( \square \).

Encourage pupils to explore the use of different symbols in their equations.
Supporting references/resources

- Use blank or empty boxes to represent variables as ‘answers’.
- Use the ‘cover-up’ method, for example using sticky notes, to stimulate discussion about substitution into functions and mappings.

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<tr>
<td>recognise, describe and recreate simple repeating patterns and sequences</td>
<td>recognise and use a simple pattern or relationship</td>
<td>use and interpret mathematical symbols and diagrams</td>
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</table>
Algebra objective

Plot the graphs of linear functions, where \( y \) is given explicitly in terms of \( x \), on paper and using ICT (Y8)

Possible barrier

Pupils cannot plot coordinates in the first quadrant.

Teaching ideas and questions

Starting from the coordinate pair \( (2, 1) \), ask:

- Which number is the ‘across’ value?
- Which number is the ‘up’ value?
- What do the numbers in a coordinate pair mean?
- Does it matter whether you use the horizontal or vertical axis first?
- What is the same/different about these two coordinate pairs: \( (2, 1) \) and \( (1, 2) \)?
  Would you plot them in the same position on a graph? Explain how you know.

- Show me:
  - how to use the scale on the axes to help you to read a coordinate that has been plotted
  - how to use the scale on the axes to help plot a coordinate accurately.

Give the coordinates of three vertices of a square and ask pupils to find the fourth:

- How do you know these coordinates are correct?
- Could you work out the coordinates of the fourth vertex without drawing the square?
  How could you do this?

Pupil activities and opportunities for mathematical talk

It is important to to be clear about the focus to be developed. There are two suggested options:

Option A: Focus on the ability to draw and accurately label axes. This is an important skill and familiarity with this supports understanding of coordinate representation.

Option B: Focus on the visualisation of a relationship between two variables. For this provide the axes and scales. However, if pupils have previously used their own symbols then, assuming that these are mathematically correct, these ought to be adopted for the labels used for this learning.

Supporting references/resources

There is a wealth of materials to support this objective on the web. Such materials can be found by entering ‘coordinates’ into an appropriate search engine.

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<td>measure and order objects</td>
<td>describe the position of objects</td>
<td>use and interpret</td>
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<tr>
<td>using direct comparison</td>
<td></td>
<td>coordinates in the</td>
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<td>first quadrant</td>
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**Algebra objective**

*plot the graphs of linear functions, where $y$ is given explicitly in terms of $x$, on paper and using ICT (Y8)*

**Possible barrier**
Pupils do not recognise that the variables (dependent and independent) can be used as numbers in a coordinate pair.

Note: Here we are linking three representations of the same mathematical relationship: an equation, a sequence, and a line on a graph. This is challenging but is worth the struggle. To fully understand the interconnections pupils will need to revisit the concept on several occasions. On the first occasion you may only ‘sow the seeds’ of this big idea.

### Teaching ideas and questions

The starting point is from the plotted coordinate pair. Ask pupils:

- *How can I explain where this point is on the graph?*
- *What makes this point different from this point?*
- *How could I describe the position of this new point to a friend on the phone?*

Ask pupils to interpret a set of, say, five points that lie on the same line. They should identify the coordinates of each point, comment on any number connections they notice and explain what each number in the coordinate pair means.

### Pupil activities and opportunities for mathematical talk

$\cdot = \square + 2$. Encourage pupils to decide on their own symbols and then follow through activities using these agreed symbols.

Record the values of $\cdot$ for various values of $\square$. This does not need to be in the form of a table; encourage pupils to choose their own way of organising the recording these results.

Ask pupils to use a pair of axes labelled $\cdot$ and $\square$ and record their results on this grid. The link between the equation and its graphical representation can be drawn out without the need for an algorithmic approach.

Encourage pupils to experiment with a variety of different symbols, using them in both their equations and for labelling the axes of their grids.

Inputs or outputs of a function machine might be coloured separately and axes coloured in a similar way.

Use ‘people geometry’, asking pupils to act as points in a coordinate system.

Make up three sets of matching cards for pupils to sort: one set showing a list or tables of coordinate pairs, another with axes showing the corresponding discrete sets of points and the third showing graphs of the corresponding linear functions.

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